Verification of an efficiency model for very large wind turbine clusters

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Abstract

In the paper an analytical model for very large arrays of wind turbines is verified against measurements. Contrary to existing engineering tools the model accounts for the wind farm's interaction with the atmospheric boundary layer. As it is often the need for offshore wind farms, the model handles a regular array-geometry with straight rows of wind turbines and equidistant spacing between units in each row and equidistant spacing between rows. The case with the flow direction being parallel to rows in a rectangular geometry is considered by defining three flow regimes. In this paper the case of the direct wake with flow within ±2.5° of the direction of the rows is considered.

Data used here are from the measurements presently being conducted at the Danish demonstration offshore wind farm at Horns Rev. The data encompass power from the wind turbine units and other relevant parameters, and flow characteristics measured by means of a number of meteorological masts situated in the vicinity of the wind farms. Results from the smaller offshore wind farm at Middelgrunden are used to provide a second case study. There are no meteorological measurements to provide the freestream but the power output from each turbine in the wind farm is available.

The model is not in a "closed form" and therefore experimental information is needed to determine model parameters. The geometries of the two wind farms are different, thus offering a range of flow conditions, which is useful in terms of model verification. Therefore, results from measurements will serve both model calibration and verification of the principles applied.

Introduction

A model for large wind farms has been developed which calculates the wind speed deficit in wind farms from wind turbine wakes based on an analytical method and encompasses flow within and downwind of the wind farm. The model is described in detail in (Frandsen et al. 2004), and in brief in section 1.1.

Here we focus on power loss in the direct wake within large offshore wind farms. This means that for flow from a narrow sector (~5°) the wake at each turbine is fully developed and we do not consider partial wake situations. Model calculations are compared with observations from the Horns Rev (Jensen 2004) and Middelgrunden offshore wind farms (Jørgensen et al. 2003) in Denmark.

While wake development in the full direct wake situation clearly leads to the greatest loss of power due to wakes in the row, it does not describe the power loss in the wind farm under all conditions. The model can handle the partial wake situation and these results will be presented in future work.

1.1 The model

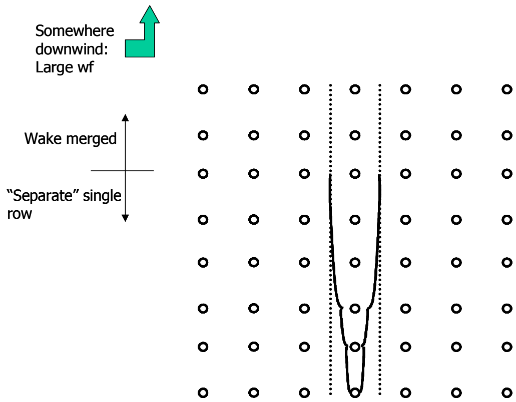

Figure 1 *Illustration of the regimes of the analytical model. The wind comes from a direction parallel to the rows.*

The model-complex links the small scale and large scale features of the flow in wind farms. The model currently handles regular array-geometry with straight rows of wind turbines and equidistant spacing between units in each row and equidistant spacing between rows. Firstly, the case with the flow direction being parallel to rows in a rectangular geometry is considered by defining three flow regimes. Situations when the flow is not in line with the main rows will be discussed in future work. Counting from the upwind end of the wind farm, the model encompasses three regimes as illustrated in Figure 1: In the first regime, the wind turbines are exposed to multiple-wake flow and an analytical link between the expansion of the multiple-wake and the asymptotic flow speed deficit are derived. The second regime materializes when the (multiple) wakes from neighbouring rows merge and the wakes can only expand vertically upward. This regime corresponds (but is not identical) to the flow after a simple roughness change of terrain. The third regime is when the wind farm is “infinitely” large and flow is in balance with the boundary layer.

2 The wind farms

2.1 Horns Rev wind farm

Horns Rev wind farm has 80 turbines laid out in 10 rows west to east with a spacing of about 7 rotor diameters (D) (560 m) between and along rows (Figure 3). The turbines are Vestas V80 2 MW with a hub-height of 70 m and a rotor diameter of 80 m. Meteorological measurements are made at three masts at the site. The nearest point to the coast is 14 km away. The data set comprising both meteorological and wind turbine parameters has only recently become available and thus far is relatively limited in terms of the number of observations.

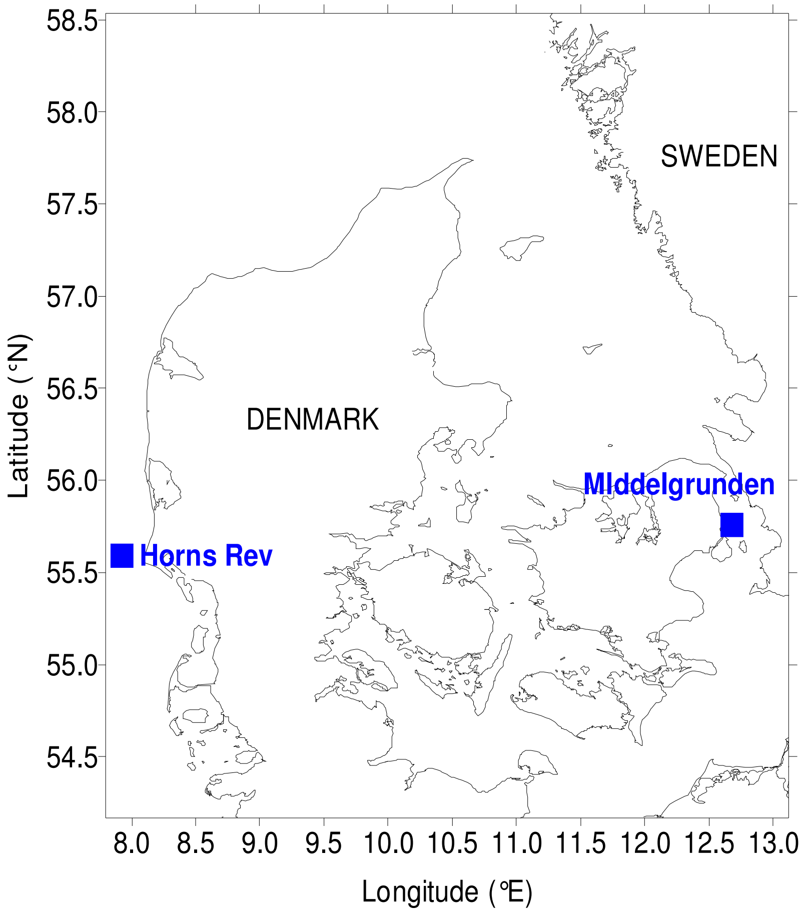

Figure 2. The location of the Horns Rev and Middelgrunden wind farms.

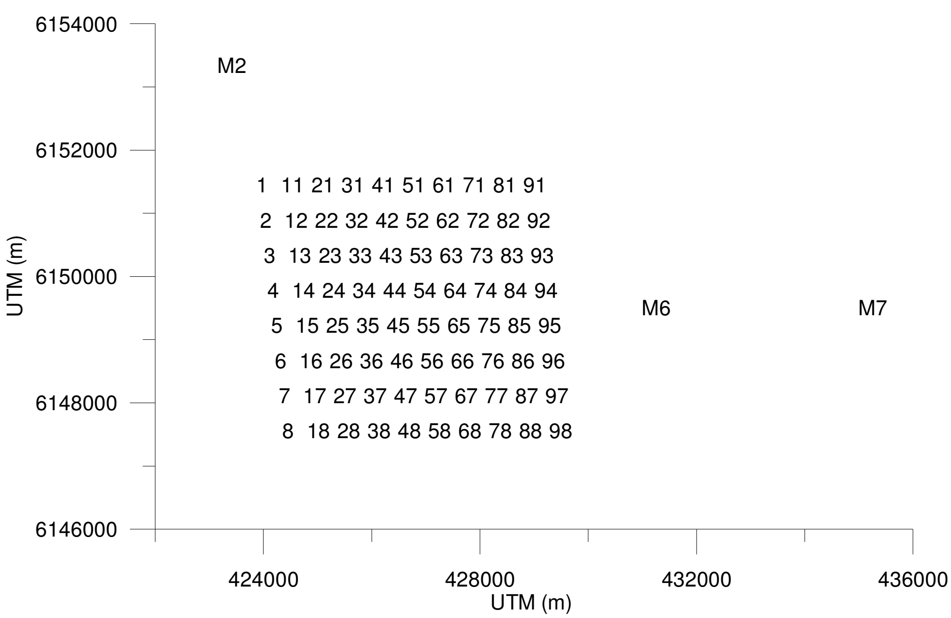

Figure 3. Layout of Horns Rev wind farm (positions in UTM zone 32). Numbers denote the wind farm while the meteorological masts are shown as M2, M6 and M7.

2.2 The Middelgrunden wind farm

The Middelgrunden wind farm consists of twenty 2 MW BONUS wind turbines with a hub height of 64 m and a rotor diameter of 76 m. The turbine array is located outside of Copenhagen and the turbines are equally spaced on a circle with a diameter of approximately 12 km (Figure 4). The spacing is 2.6 D. The distance to the coast is just over 2 km.

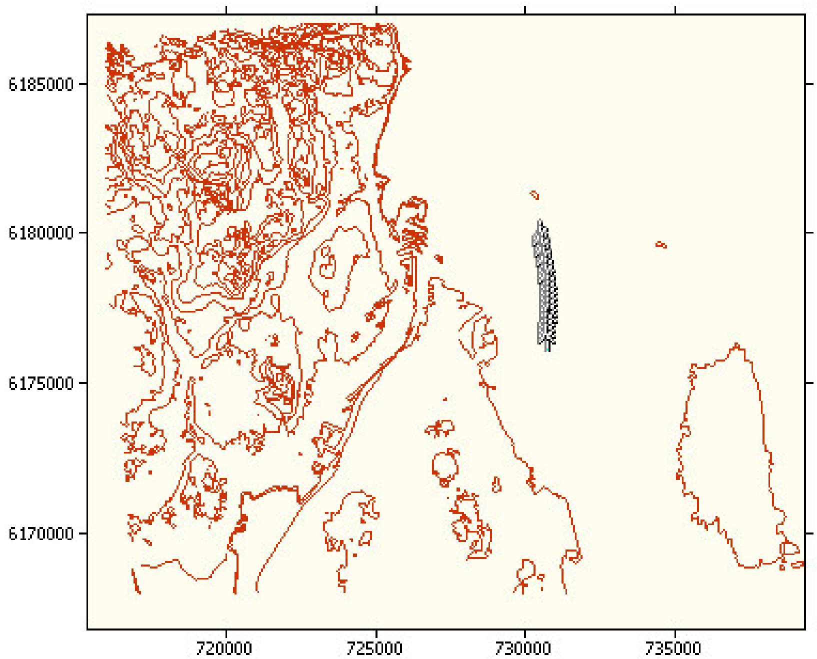

Figure 4. The layout of the Middelgrunden wind farm. Lines shown are height contours and the coordinates are in UTM zone 32.

3 Direct wake observations

3.1 Horns Rev

In order to classify the wake power loss along the row a number of conditions must be fulfilled. The wake response is strongly dependent on wind speed. Here we focus on wakes for freestream wind speeds of 8-10 m/s where the value for the thrust coefficient is high and relatively constant. The wake direction is 269±2° at Horns Rev. A centre row (row 5) is chosen to avoid edge effects and all turbines in the row must be operating in order for the observation to be included. As shown in Figure 5, the initial drop in power from the first turbine in the freestream wind is significant (down to around 0.65) and the subsequent power loss is relatively small (~2%). Using a narrow direction sector gives a relatively limited data set. This adds to the uncertainty in the calculations.

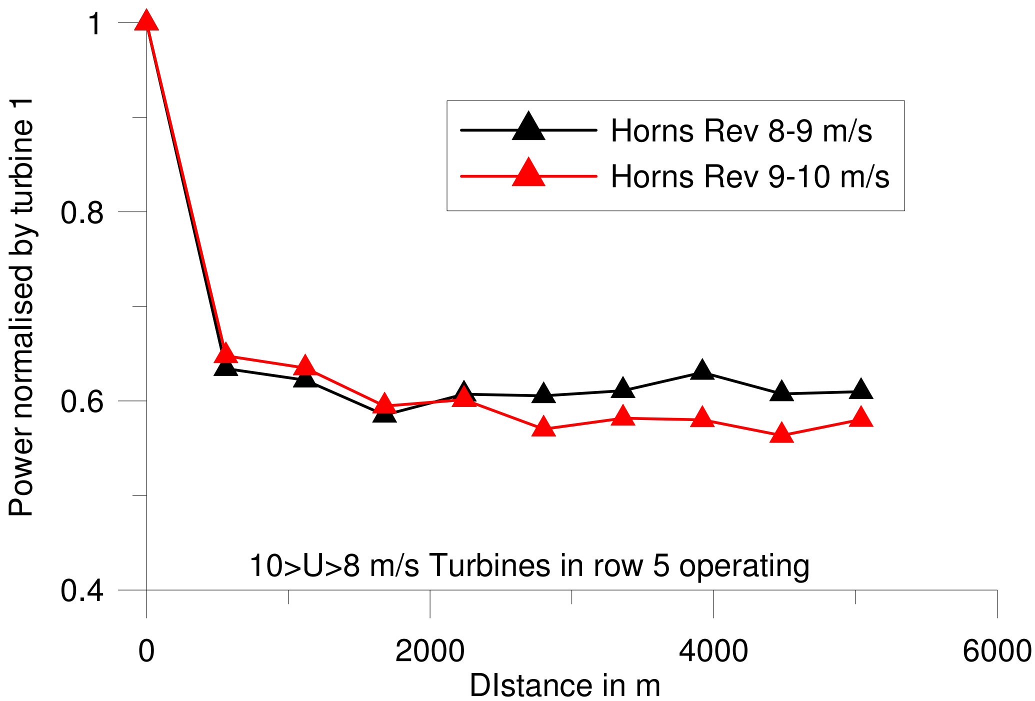

Figure 5. Comparison of the power loss along a row of wind turbines for a direct wake at Horns Rev for freestream wind speeds 8-10 m/s shown by distance from the first wind turbine from observations.

It is important to recall:

1) wake losses are highly wind speed dependent
2) the direct wake is the most extreme case in terms of power loss in wind farms.

3.2 Middelgrunden

It is difficult to specify the direct wake at Middelgrunden due to the unusual layout of the wind farm and the lack of independent meteorological measurements to provide freestream conditions. There is some uncertainty in defining the freestream direction by the yaw angle of the turbine. The wind farm data are taken from (Jørgensen et al. 2003) where the wind speeds were calculated from the power output/power curve and analysed in 2° sectors (Figure 6). Here we focus on a narrow sector 3-5° close to north as measured at the first (north) turbine. However, data analysis suggests that this is the direct wake direction at least in the first part of the row. After the initial decrease in wind speed/power output at turbine 2 the wind speed/power output begins to increase after turbine 10. There are a number of possible reasons:

1) gradient in ambient wind speed caused by topography
2) misalignment of the wind turbines with respect to the assumed freestream wind direction
3) complex wake interactions.

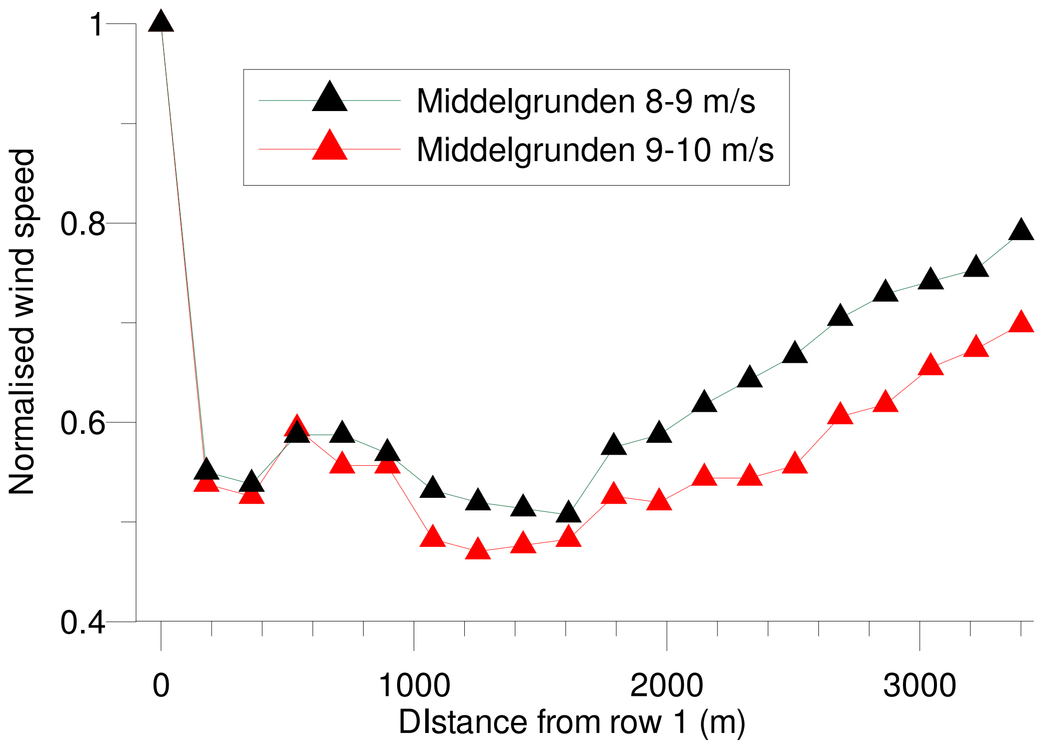

Figure 6. Normalised wind speed at Middelgrunden for direction 3-5° from observations.

4 Modelling

4.1 Single/multiple wake modelling

The fundamentals of the analytical model for large wind farms are given in Frandsen et al. (2005). In brief the expansion of the wake behind a wind turbine is given by the general solution:

$$D_x = \left(\beta^{n/2} + \alpha . s\right)^{1/n} D_{0,} \quad s = x / D_0$$

where the solution for n has been suggested as 3 by e.g. Schlichting (1968).

α is the decay constant which is related to the thrust coefficient C_T and β is the initial wake expansion, also being calculated from the thrust coefficient:

$$\beta = \frac{0.5 * (1 + \sqrt{1 - CT}\,)}{\sqrt{1 - CT}\,)}$$

Figure 8 shows results for the single/multiple wake model which does not account for turbine wake interactions with the ground. By adjusting parameters in the model it is possible to get a good fit to the observed data. Notably, the exponent parameter was chosen as n=1, which choice provides the excellent fit for the first few units. At the end of the row, the model recovers too well, which is expected: with the chosen way of modelling the asymptotic value of n for increasing wind turbine number must be ½ to ensure a non-vanishing wind speed deficit.

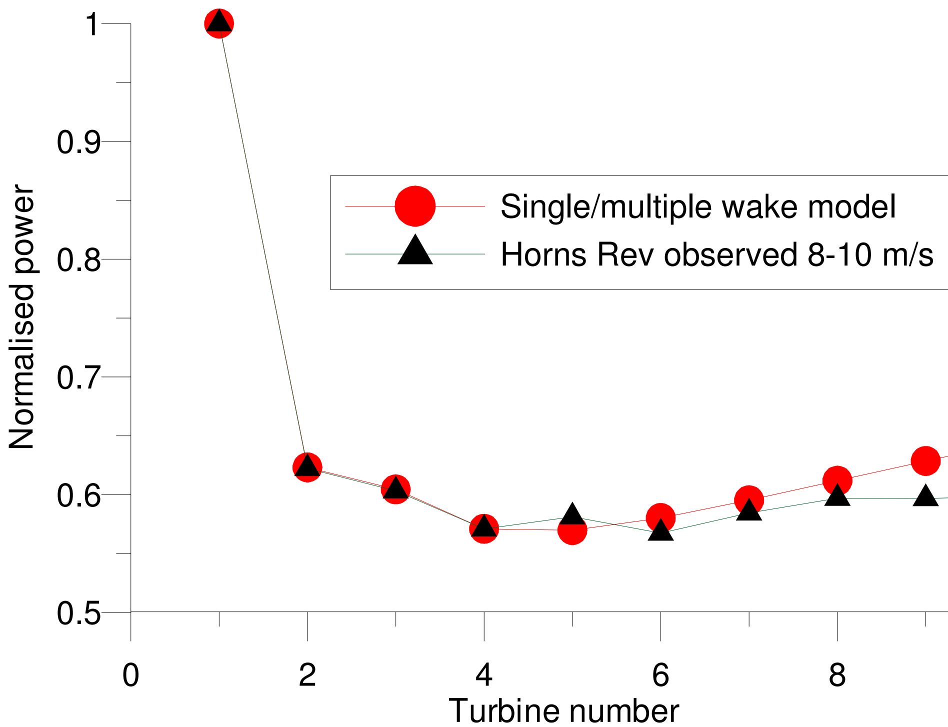

Figure 8. Comparison of single wake model with observations for Horns Rev.

4.2 The analytical model

The major difference between the 'full' analytical model and the single/multiple wake model described in section 5.1 is that the full model accounts for turbine wake interactions with the ground. This is done by calculating the momentum deficit behind each turbine and keeping this constant. The wind speed deficit gradually becomes smaller with the distance from the turbine because the wake area is expanding. The model has an additional parameter which controls the merging of wakes in the downwind direction. Here the distance at which the inner or most immediate wake is merged with the wind farm or outer wake is set according to the ratio of wind speeds in the two wakes. For Horns Rev the ratio is set to 0.6. This is difficult to justify at this point, other than it gives a reasonable fit to the data. Clearly there is still some work to do on the parameterisations used in the model which have to be rationalised by further data analysis (see section 4.1).

4.2.1 Horns Rev

The analytical model as described in Frandsen et al. (2005) was applied to the case at Horns Rev. Using the original model version with α= 0.175*4 and $\beta_* = \beta^{3/2}$ where the thrust coefficient was set according to the wind speed gives good agreement with observations except at turbine 2 where the predicted wind speed is too low. This suggests the wake recovery is initially too slow (Figure 9). There are still some issues in the model related to wake merging which require further work.

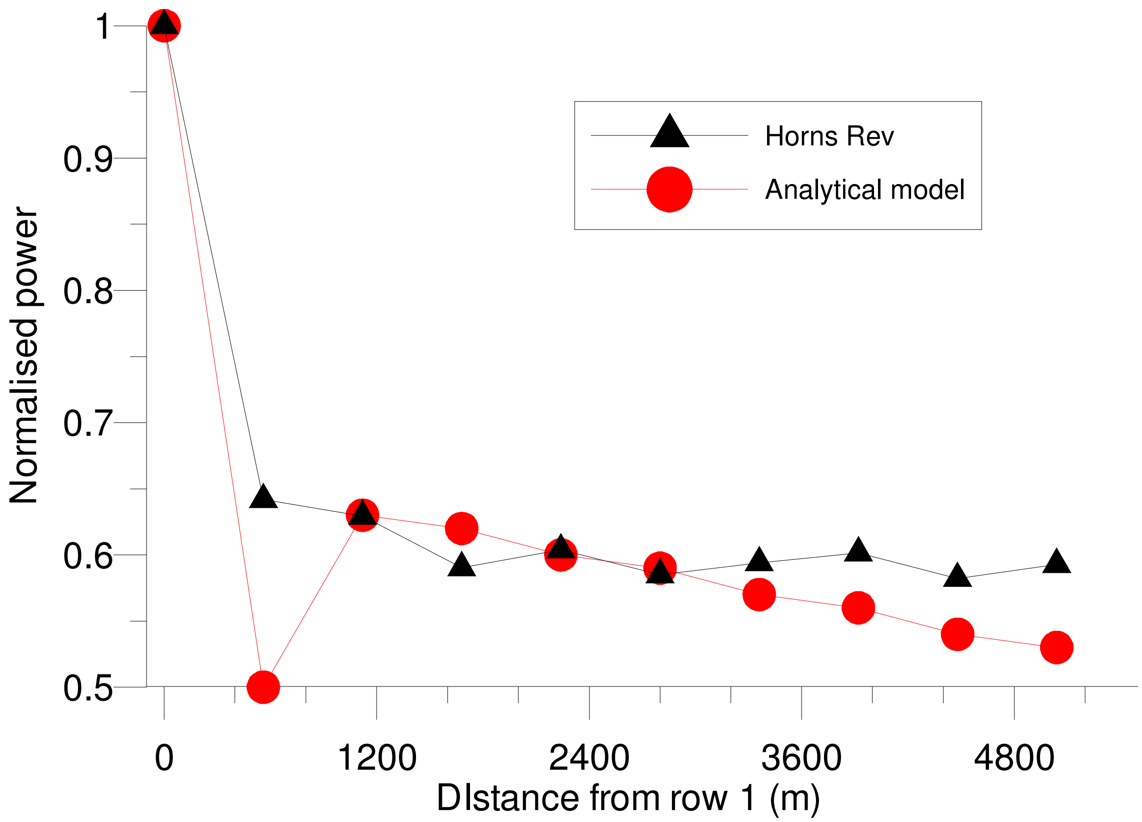

Figure 9. Results showing the normalised power output for a central row at Horns Rev.

4.2.2 Middelgrunden

The analytical model was run for the 4° direction at Middelgrunden for a mean wind speed of 8-10 m/s. As shown in Figure 10 the model captures the initial wind speed drop but is unable to reproduce the subsequent recovery. However this is a complex case in terms of layout which needs further research. Note the downwind merging ratio is set here to 0.7.

5 Summary

Power loss in direct wakes has been examined using data from two offshore wind farms. Based on two large wind farms, albeit with relatively limited number of observations, the direct wake case shows that:

- In a direct wake situation (±2.5°) the average power of the subsequent turbines in the row is of the order 65% of the freestream turbine for wind speeds of 8-10 m/s. The power loss continues down the row but the difference in power output between the second and last turbine is moderate – of the order 2-5%. Note that this is the most extreme case and results for the non-direct wakes will be presented in later work.
- It is possible to reproduce the observed wind speed/power output at subsequent turbines in the row using either the simple model for wake expansion/interaction or the full model where the wakes interact downwind, laterally and with the ground keeping the momentum deficit constant. However, there are a number of parameters in the model and further data analysis is necessary in order that these can be used as calibration.

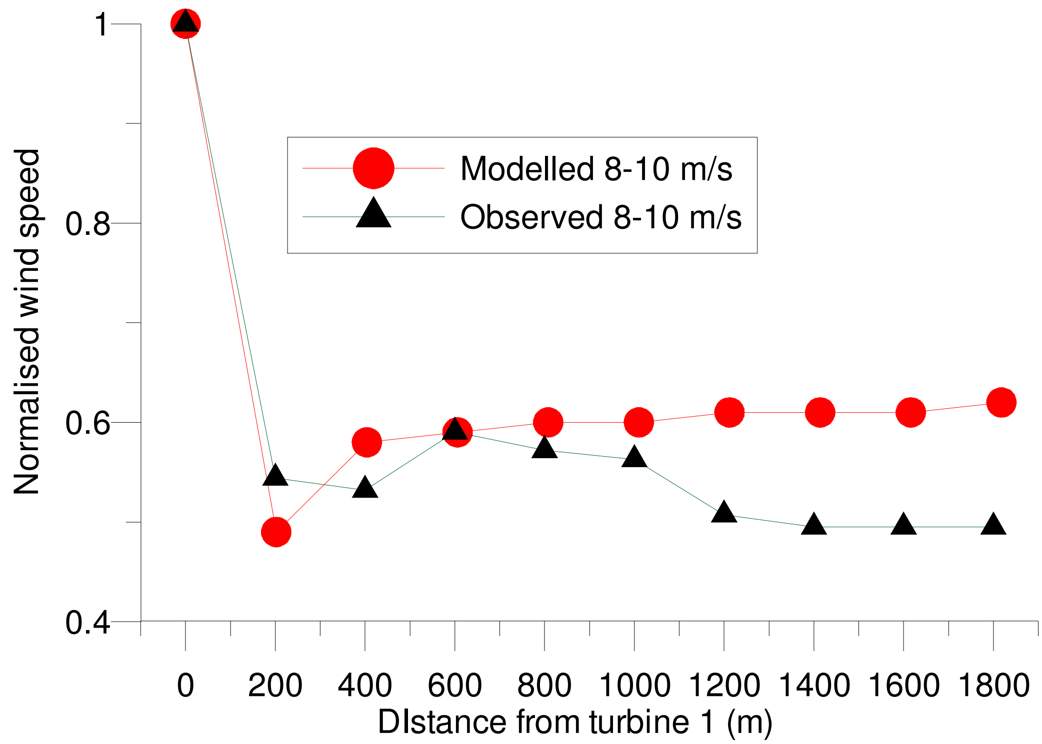

Figure 10. Results showing the normalised wind speed for a northerly direction at Middelgrunden.

6 Acknowledgements

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